2013

NAME: KEY

Test: Gravity

Some equations you may need:

$$F = G\frac{m_1 m_2}{r^2} \quad \frac{T^2}{R^3} = \frac{4\pi^2}{GM} \quad v_e = \sqrt{\frac{2GM}{r}} \quad U = -\frac{Gm_1 m_2}{r} \quad E = -\frac{GmM}{2R}$$

$$F_c = \frac{mv^2}{r} \quad K = \frac{1}{2}mv^2 \quad L = I\omega \quad \vec{\tau} = \vec{r} \times \vec{F} \quad I = \sum mr^2 = \int r^2\, dm$$

add $e = \frac{c}{R}$ $T = \frac{2\pi}{\omega}$

Some constants you may need:

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$ $M_{earth} = 6 \times 10^{24}$ kg $R_{earth} = 6.4 \times 10^6$ m $D_{earth\text{-}sun} = 1.5 \times 10^{11}$ m

$M_{sun} = 2 \times 10^{30}$ kg $R_{sun} = 7 \times 10^8$ m $M_{moon} = 7.35 \times 10^{22}$ kg $R_{moon} = 1.8 \times 10^6$ m

Multiple Choice: *Choose the letter of the best answer. 3 points each.*

1. D In the formulas above, G is a
   a. depends on the local value of g.
   b. is used only when Earth is one of the two masses.
   c. is greatest at the surface of Earth.
   d. is a universal constant of nature.
   e. is related to the Sun in the same way that g is related to Earth.

$\frac{1/100}{1/4^2} = \frac{16}{100} \cdot 600 = 96$

2. C The mass of a hypothetical planet is 1/100 that of Earth and its radius is 1/4 that of Earth. If a person weighs 600 N on Earth, what would he weigh on this planet?
   a. 24 N. b. 48 N. c. 96 N. d. 192 N. e. 600 N.

Problems 3 and 4 refer to the following:
Imagine you can put a rock anywhere you like - either above the earth or in the earth in a narrow tunnel dug down to the center of the earth. R_e is the radius of the earth.

3. C At what radius from the center could you place the rock inside the earth so that it weighed half as much?
   a. 0. b. $\frac{1}{4}R_e$ c. $\frac{1}{2}R_e$ d. $\frac{\sqrt{2}}{2}R_e$ e. none of those.

4. D At what radius from the center could you place the rock outside the earth so that it weighed half as much?
   a. 0. b. $4R_e$ c. $2R_e$ d. $\sqrt{2}R_e$ e. none of those.

5. B What does it mean for two objects to have 0 gravitational potential energy?
   a. The two objects are touching each other. ✗
   b. The two objects are infinitely far away. ✓
   c. The two objects are orbitting each other with circular orbits.
   d. Huh? The situation is impossible. ✗

6. C What is the minimum length of a day the earth could have that would allow the earth to just barely stay together?
   a. 0 s. b. 294 s. c. 5084 s. d. 86,400 s. e. impossible to say

7. C Two identical objects (each mass m) are seperated by a distance of r. How much work would it take to triple the distance between between them?
   a. $\frac{Gm^2}{r}$ b. $\frac{Gm^2}{3r}$ c. $\frac{2Gm^2}{3r}$ d. $\frac{4Gm^2}{3r}$ e. $\frac{3Gm^2}{r}$

8. A

$-\frac{1}{r} + W = -\frac{1}{3r}$

9. C

$3 = \frac{1}{r} - \frac{1}{3r} = \frac{3-1}{3r} = \frac{2}{3}$

$\frac{1}{2}m\left(2\frac{GM}{r}\right)$

10 @ 3 = 30

6 @ 12 = $\frac{72}{100}$

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8. A If a 1500 kg object were to "fall" into the sun from infinitely far away, how fast would it be moving when it hit the sun?
   a. 617,000 m/s. b. 437,000 m/s. c. 309,000 m/s. d. 0 m/s. e. ∞.

9. C What would happen to your weight if the earth were to somehow double in mass and radius?
   a. It would not change at all because your mass did not change.
   b. It would be four times greater.
   c. It would be 1/2 as much as before.
   d. It would be 1/4 as much as before.
   e. It would be twice as much as before.

$\frac{\times 2}{(\times 2)^2} = \frac{2}{4} = \frac{1}{2}$

9@3=27 6@12=72 84 → 90 100

Problem Solving: *Show all work.* 12@

11. A mass M is split into two parts, m and $M - m$, which are then separated by a certian distance. What ratio m/M maximizes the magnitude of the gravitational force between the parts?

$$F = G\frac{m(M-m)}{r^2} = \frac{G}{r^2}(Mm - m^2)$$

no reason −3

$$\frac{dF}{dm} = \frac{G}{r^2}(M - 2m) = 0$$

Just F= −8

$$M - 2m = 0$$

$$M = 2m \rightarrow \boxed{m/M = \frac{1}{2}}$$

12. How fast would you have to throw something from the ground so that it reached a height of 2000 km above the surface of the earth? (Ignore air resistance.)

$R_f = 8.4\times10^6$ m

$$U_i + K = U_f$$

$$-G\frac{mM}{R} + \frac{1}{2}mv^2 = -G\frac{mM}{(R + 2000\text{ km})}$$

$$v^2 = \frac{2GM}{R} - \frac{2GM}{R + 2{,}000{,}000}$$

$$v^2 = 2GM\left[\frac{1}{R} - \frac{1}{R + 2000000}\right]$$

$$= 2(6.67\times10^{-11})(6\times10^{24})\left[\frac{1}{6.4\times10^6} - \frac{1}{8.4\times10^6}\right]$$

$$v^2 = (8\times10^{14})(3.72\times10^{-8})$$

$$v^2 = 29.8\times10^6$$

$$\boxed{v = 5457}$$

v_e −10, −6, K=0

13. The acceleration due to gravity on a random planet is 7.5 m/s². If the planet has a uniform density of 4000 kg/m³, what is the escape velocity from the surface of the planet?

$$g = G\frac{M}{r^2} \qquad \rho = \frac{M}{\frac{4}{3}\pi r^3} \rightarrow \therefore M = (4000)(\tfrac{4}{3}\pi)(6.7\times10^6)^3$$

$$M = 5.06\times10^{24}$$

too many! $M = 3000\pi r^3$!!!

$$g = G\frac{\rho\frac{4}{3}\pi r^3}{r^2} = \frac{4}{3}\pi\rho G r \rightarrow 7.5 = \frac{4}{3}\pi(4000)(6.67\times10^{-11})R$$

$$R = 6.7\times10^6$$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2\cdot G\cdot(5.06\times10^{24})}{6.7\times10^6}} = \boxed{10{,}033 \text{ m/s}}$$

−3

$$v_e = \sqrt{2gR} = \sqrt{3g^2/2\pi\rho G} \leftarrow r = \frac{3g}{4\pi\rho G}$$

a

24

Test: Gravity

14. A 1000 kg asteroid in orbit around the sun has a total energy of -9×10^{11} J and an eccentricity of 0.25. What is the slowest speed the asteroid will have in its orbit around the sun?

$E = -\frac{GmM}{2R} = -G\frac{(1000)(2\times10^{30})}{2R} = -9\times10^{11}$

$R = 7.41 \times 10^{10}$ m $a = 9.26 \times 10^{10}$ m

$= .494$ AU

$a = R + c$

$a = R + eR$

$a = .494(1 + .25)$ $a = .618$ AU

$E = K + U$

$-9\times10^{11} = \frac{1}{2}(1000)v^2 - G\frac{(1000)(2\times10^{30})}{(9.26\times10^{10})}$

$-9\times10^{11} = 500v^2 - 1.44\times10^{12}$

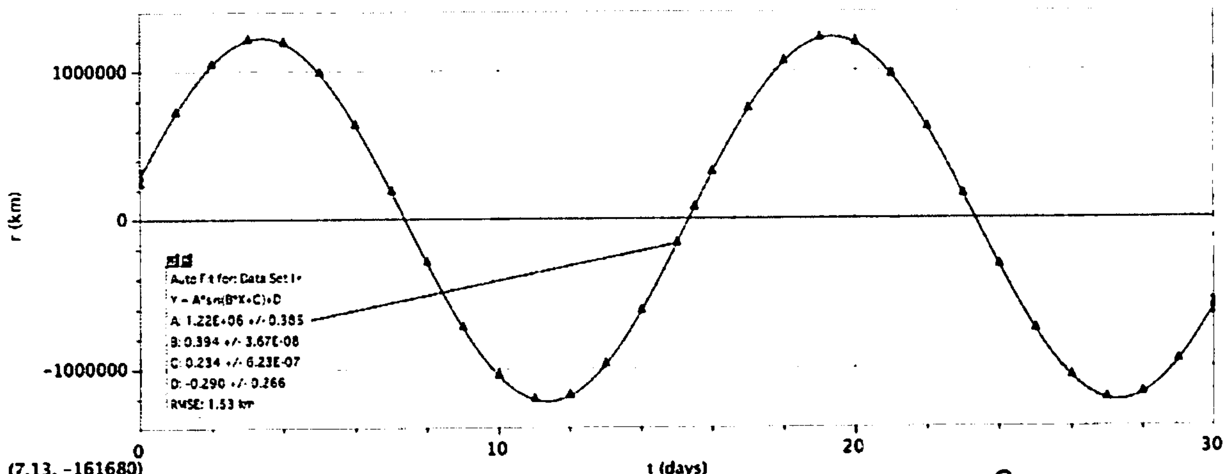
$v^2 = 1.09\times10^9$

$v = 32{,}900$ m/s

no R −4

15. Titan (a moon of Saturn) was observed over many days. A graph of its distance from the center of Saturn (in km) vs time (in days) is shown, with the best fit function shown. (radians were used, not degrees.) What is the mass of Saturn?

Titan Orbital Data



$R = 1.22\times10^6$ km $= 1.22\times10^9$ m

$T = \frac{2\pi}{\omega} = \frac{2\pi}{.394} = 15.95$ days $= 1.38\times10^6$ s

$\frac{T^2}{R^3} = \frac{4\pi^2}{GM}$

$M = \frac{4\pi^2 R^3}{G T^2} = \frac{4\pi^2(1.22\times10^9)^3}{G(1.38\times10^6)^2}$

$M = 5.66\times10^{26}$ kg

km's days −3

(km's) −2

16. Derive one of Kepler's Laws. (You can assume a circular orbit for Kepler's 3rd Law, but not for the 2nd Law.) Please use a little English to explain what you are doing.

($\Sigma\tau = 0$ ∴ L = constant) −3

just $\frac{4\pi^2}{GM}$ −2

Non-planar ball −4